

HOMEWORK # 9

The Lotka-Volterra model of competition is often described as a model of resource competition, because the presence of the competitor suppresses abundance below the species' carrying capacity, as if the competitor was using up resources. However, species may also compete for mates, in a form of competition known as "reproductive interference." This type of competition is especially common among closely related species, where members of each species may mate with the other species. Assuming these matings do not produce viable offspring (because the embryos fail to develop, because the offspring do not survive to reproductive maturity, or because the offspring are sterile), the presence of the competitor will reduce the birth rate of each species. A model for reproductive interference between two species, N_1 and N_2 , is

$$\frac{dN_1}{dt} = \left(b_1 \left(\frac{N_1}{N_1 + c_{12}N_2} \right) - d_1 \right) N_1 - h_1 N_1^2, \quad (1)$$

$$\frac{dN_2}{dt} = \left(b_2 \left(\frac{N_2}{N_2 + c_{21}N_1} \right) - d_2 \right) N_2 - h_2 N_2^2. \quad (2)$$

- (1) Explain each parameter of the dN_1/dt equation in biological terms. You can assume that $b_i > d_i$ and that c_{ij} varies between 0 and 1. (If you are having trouble, figure out what the equilibrium of the population would be in the absence of N_2 .)
- (2) What are the equations for the N_1 and N_2 nullclines?
- (3) Draw two sets of N_1 and N_2 coordinate planes. Draw the N_1 nullclines on one coordinate plane, and draw the N_2 nullclines on the other plane. Label all intersections with the nullclines and either axis. (If you are having trouble figuring out what the clines look like, specify parameter values and use R or Wolfram Alpha to draw them for you. If you use this approach, you should try several different sets of parameter values to confirm that the shape of the curve is not affected by your choice.)
- (4) Combine the nullclines on a single coordinate plane. Create all possible nullcline configurations. For each configuration, clearly identify the equilibria. Draw the direction of the vector field in each region of the phase plane. Clearly identify whether the equilibria are stable or unstable.
- (5) Is reproductive interference more or less likely to allow two species to coexist than resource competition, as modeled by the Lotka-Volterra competition model? Justify your answer using the results from the preceding question, especially as they pertain to the stability of any equilibria.