Many marine turtle populations are threatened with extinction due to human activities. However, in recent years, considerable effort has been put into their conservation. Most of these efforts have focused on the egg stage, protecting nesting sites and the eggs laid therein. Loggerhead turtles (*Caretta caretta*) are of the best studied marine sea turtles, as many of their largest and most important nesting sites are in the southeastern United States. The life cycle of the loggerhead sea turtle can be subdivided into seven biologically distinct stages:

(1) 1st year (eggs and hatchlings),
(2) small juveniles,
(3) large juveniles,
(4) subadults,
(5) novice breeders,
(6) 1st-year remigrants,
(7) mature breeders.

As is typical for stage-structured population models, only the demography of females is considered. Novice breeders are females breeding for the first time. 1st-year remigrants are females breeding for the second time - the name of this stage alludes to the fact that loggerheads return to the same beaches, year after year, to breed. The distinction between stages 5-7 is primarily in fecundity, not survivorship. The transition matrix for loggerhead sea turtles is

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 127 & 4 & 80 \\ 0.6747 & 0.7370 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0486 & 0.6610 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0147 & 0.6907 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0518 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8091 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8091 & 0.8089 \end{pmatrix} \tag{1}$$

(1) Draw the life cycle diagram for this population.
(2) Using R (or Wolfram Alpha, Sage, Mathematica, or Maple), calculate the dominant eigenvalue. Is the population growing or declining?
(3) What is the stable stage distribution (given by the right eigenvector associated with the dominant eigenvalue)? Remember to rescale so the elements in this vector sum to one. Which stage makes up the majority of the population at the stable age distribution? If you have done this calculation correctly, you should find that a stage *other than* the first stage makes up the majority of the population - explain why this is the case.

(4) What are the reproductive values for each stage (the left eigenvector associated with the dominant eigenvalue)? Remember to rescale to the first stage has reproductive value equal to one.
(5) Using the values you have found for the right and left eigenvectors, which would have a larger positive impact on population growth: increasing the probability of surviving the first stage and transitioning to the second stage, or increasing the probability of surviving and remaining in the second stage?
(6) What is the dominant eigenvalue if the probability of surviving the first stage is increased to 1? What is the implication of this result for conservation strategies targeting the egg stage alone?
(7) What is the dominant eigenvalue if the probability of survival for each of the ocean-going stages (stages 2-7) is increased by 0.1 (that is, each survivorship goes up by 10%; for stages 2-4, increase the probability of surviving but remaining in the stage)?