HOMEWORK 7

(1) (2 pts) Consider a population in which individuals can be classified as "juveniles" and "adults", where juveniles are not yet reproductively mature. Use $J(t)$ and $A(t)$ to denote the number of juveniles and adults at time t. Suppose that adults produce b juvenile offspring that survive to the next timestep, and that adults survive to the next timestep with probability p_a. Juveniles survive to the next timestep and remain juveniles with probability p_j; they survive and mature to the adult stage with probability p_{aj}. Derive recursion equations for this system and write this system in matrix notation. Assuming $p_a = 0.4$, $p_j = 0.2$, and $p_{aj} = 0.1$, how large does b have to be to guarantee that the extinction equilibrium is unstable?

(2) (3 pts) In class we introduced the general age-structured transition matrix $\mathbf{M}$:

$$\mathbf{M} = \begin{pmatrix} F_1 & F_2 & F_3 & \dots & F_{x-1} & F_x \\ P_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & P_2 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & P_{x-1} & 0 \end{pmatrix} \tag{1}$$

We showed that the interpretation of the parameters F_i and P_i of this matrix depend on whether the population size within each class is estimated just before breeding, or just after (i.e., a pre-breeding versus a post-breeding census). Regardless of how a population is typically censused, the best way to estimate the parameters nedessary to calculate the F_i and P_i values is to follow a cohort of individuals from the population through their entire life. That is, track a set of individuals born during the same breeding season from birth until every individual has died. This data is often summarized in a "cohort life table" giving the number of individuals alive at each age and giving the total reproductive output of these individuals. Using the information in the cohort life table below for the perennial grass *Poa annua*, calculate the age-specific survivorship (p_i) and fecundity (m_i). Then find $\mathbf{M}$ for both the pre-breeding census case and the post-breeding census case.

Age	Number alive	Total seed production
0	843	0
1	722	300
2	527	620
3	316	430
4	144	210
5	54	60
6	15	30
7	3	10
8	0	0

(3) (5 pts) A scientist studying a colony of mice finds that they produce, on average, one surviving daughter per female in the first year of life and eight in the second year of life. She also finds that they have only a 25% chance of surviving to the second year and do not survive beyond that point. She constructs the following transition matrix for the population:

$$\mathbf{M} = \begin{pmatrix} 1 & 8 \\ 0.25 & 0 \end{pmatrix} \tag{2}$$

(a) Find the eigenvalues and right eigenvectors $\vec{u}$ of $\mathbf{M}$. (b) Using your answer in (a), what is the long-term growth rate and the proportion of the population in each age class at the stable age distribution? (c) Write a new matrix $\mathbf{L}$ containing the eigenvectors of $\mathbf{M}$ as columns. Put the eigenvector associated with the dominant eigenvalue in the first column of $\mathbf{L}$. Calculate the inverse matrix $\mathbf{L}^{-1}$. The first row gives the *left* eigenvector, $\vec{v}$, associated with the leading eigenvalue. (You can verify that $\vec{v}$ is a left eigenvector by showing that $\vec{v}^T\mathbf{M} = \vec{v}^T\lambda_1$, where λ_1 is the leading eigenvalue and $\vec{v}^T$ is the left eigenvector written in row form (i.e., the transpose of $\vec{v}$ written in column format).) (d) We can approximate the dynamics of the mouse population using

$$\vec{n}(t) \approx \lambda_1^t \vec{u}(\vec{v}^T \vec{n}(0)), \tag{3}$$

where $\vec{u}$ is the right eigenvector (written in column format) and $\vec{v}^T$ is the left eigenvector (written in row format) associated with the leading eigenvalue λ_1. With a population that begins with 10 mice, all in the first age class, what is the approximate total population size after two years? After 10 years? (e) Compare your predictions from (d) to the actual values found by iterating the transition matrix, which predicts that the total population size is 32.5 at $t = 2$ and is 7682.5 at $t = 10$. Explain why the error is proportionately worse at $t = 2$ than $t = 10$. (f) Would the population be larger if it had started with 10 mice in the second age class? Compose an answer using information from the left eigenvector.