1. $$p(t+1) = \frac{(1-\mu)\,p(t)W_A}{p(t)W_A + (1-p(t))W_a}$$

$$\hat{p} = \frac{(1-\mu)\hat{p}W_A}{\hat{p}W_A + (1-\hat{p})W_a}$$

$$\hat{p}W_A + (1-p)W_a = (1-\mu)W_A$$

$$\hat{p}(W_A - W_a) = (1-\mu)W_A - W_a$$

$$\hat{p} = \frac{(1-\mu)W_A - W_a}{W_A - W_a}$$ is an equilibrium.

$\hat{p} = 0$ is also an equilibrium.

Letting $p(t+1) = f(p(t))$, stability is determined by $\left.\frac{df}{dp}\right|_{p=\hat{p}}$.

$$\frac{df}{dp} = \frac{(1-\mu)W_A(pW_A + (1-p)W_a) - (W_A - W_a)(1-\mu)pW_A}{(pW_A + (1-p)W_a)^2}$$

$$\frac{df}{dp} = \frac{(1-\mu)W_A(p(W_A - W_a) + W_a) - (1-\mu)W_A\,p(W_A - W_a)}{(pW_A - pW_a + W_a)^2}$$

$$\frac{df}{dp} = \frac{(1-\mu)W_A W_a}{(p(W_A - W_a) + W_a)^2}$$

$$\left.\frac{df}{dp}\right|_{p=0} = \frac{(1-\mu)W_A W_a}{W_a^2} = \frac{(1-\mu)W_A}{W_a}$$

$$\left.\frac{df}{dp}\right|_{p=\frac{(1-\mu)W_A - W_a}{W_A - W_a}} = \frac{(1-\mu)W_A W_a}{((1-\mu)W_A - W_a + W_a)^2} = \frac{W_a}{(1-\mu)W_A}$$

Notice that the stability conditions are inverses. Thus, if $\left.\frac{df}{dp}\right|_{p=0} < 1$, so $\hat{p} = 0$ is stable, $\hat{p} = \frac{(1-\mu)W_A - W_a}{W_A - W_a}$ must be unstable, and vice versa.

2. One way to understand how to derive the genotype frequencies is the following: imagine the population of gametes as a pool, with the frequency of the A allele among those gametes equal to $p(t)$. Then the probability of pulling a A gamete at random is $p(t)$. That gamete will mate randomly with probability $(1-f)$, and will inbreed with probability f. If it mates randomly, the probability that the randomly chosen gamete is also A is $p(t)$. If it inbreeds, it mates with another A gamete with probability 1. Thus, the probability of producing AA zygotes is

$$P(AA) = p \cdot (1-f) \cdot p + p \cdot f \cdot 1 = (1-f)p^2 + fp.$$

By extending this reasoning,

$$P(Aa) = 2(1-f)p(1-p)$$

$$P(aa) = (1-f)(1-p)^2 + f(1-p).$$

To derive the recursion, consider first how many A gametes are produced by AA and Aa individuals. AA individuals have fitness W_{AA} and produce 2 A gametes, so the total number of A gametes from AA individuals is

$$2((1-f)p^2 + fp)W_{AA}.$$

Each Aa individual produces only 1 A gamete, so the total from Aa individuals is

$$2(1-f)p(1-p)W_{Aa}.$$

The total number of gametes produced is

$$2\left(\left((1-f)p^2+fp\right)W_{AA}+2\cdot 2(1-f)p(1-p)W_{Aa}+2\left((1-f)(1-p)^2+f(1-p)\right)W_{aa}\right)$$

Thus, the frequency of A allele in the next generation is

$$p(t+1) = \frac{p W_A}{\overline{W}}, \text{ where}$$

$$W_A = \left((1-f)p+f\right)W_{AA} + (1-f)(1-p)W_{Aa}$$

$$\overline{W} = \left((1-f)p^2+fp\right)W_{AA} + 2(1-f)p(1-p)W_{Aa} + \left((1-f)(1-p)^2+f(1-p)\right)W_{aa}$$

3. Letting $p(t+1)=f(p(t))$, to determine the stability of $\hat{p}=0$, we need to evaluate $\left.\frac{df}{dp}\right|_{p=0}$. Writing

$$f(p(t)) = \frac{N(p(t))}{D(p(t))}, \quad \left.\frac{df}{dp}\right|_{p=0} = \frac{N'(0)\cdot D(0) - N(0)\,D'(0)}{D(0)^2}$$

$$N'(0) = 2pW_{AA} + \left(1-\frac{f}{2}\right)(1-p)W_{Aa} - \left(1-\frac{f}{2}\right)pW_{Aa}$$

$$= \left(1-\frac{f}{2}\right)W_{Aa}$$

$$D(0) = (1-f)W_{aa} + fW_{aa} = W_{aa}$$

Because $N(0)=0$, no need to compute $D'(0)$.

$$\left.\frac{df}{dp}\right|_{p=0} = \frac{\left(1-\frac{f}{2}\right)W_{Aa}W_{aa}}{W_{aa}^2} = \frac{\left(1-\frac{f}{2}\right)W_{Aa}}{W_{aa}}.$$

This is stable if $\left(1-\frac{f}{2}\right)\frac{W_{Aa}}{W_{aa}} < 1$.

Thus if $1-\frac{f}{2} < \frac{W_{aa}}{W_{Aa}} \Rightarrow f > 2\left(1-\frac{W_{aa}}{W_{Aa}}\right)$. Notice that since $W_{aa} < W_{Aa}$, $W_{aa}/W_{Aa} < 1$. Only if W_{aa} is relatively close to W_{Aa} can inbreeding lead $\hat{p}=0$ to be stable. In particular if $W_{aa}/W_{Aa} < 0.5$, no amount of inbreeding can lead to a stable equilibrium.