1. (3 pts) In class, we derived a model of natural selection when selection happens during the diploid phase of the life cycle. A simpler model of natural selection is one where selection happens during the haploid (gamete) stage. In this case, we can assign fitnesses to each allele directly, as W_A and W_a, and we can effectively ignore the diploid stage. A model for such a system is

$$p(t+1) = \frac{p(t)\, W_A}{p(t)\, W_A + (1-p(t))\, W_a}$$

Assume that mutation is occuring, but only in one direction, from $A \to a$, and that a frequency μ of A gametes mutate to a. Then the model becomes

$$p(t+1) = (1-\mu)\, \frac{p(t)\, W_A}{p(t)\, W_A + (1-p(t))\, W_a}$$

Find the two equilibria of this system and determine the stability conditions for each.

2. (4 pts) Mating within a population is often not random. In particular, if the population is very spatially structured (so that subpopulations seldom come in contact with one another) or if kin (closely-related individuals) prefer to mate with one another, then gametes carrying the same alleles unite more often than one would expect under random mating, a phenomenon known as *inbreeding*. A simple model of inbreeding is the following: each gamete, randomly chosen from the population, will mate with another randomly chosen gamete with probability $1-f$, and will mate with a gamete carrying the same allele with probability f. The parameter f is known as the *inbreeding coefficient*. Assuming that the frequency of the A allele in the entire population is $p(t)$, convince yourself that the probability of producing an AA zygote under this model is $(1-f)\, p(t)^2 + f\, p(t)$.

(a) What are the probabilities of producing an Aa zygote and an aa zygote? (Hint: a good check on your answer is to make sure that the probabilities of producing AA, Aa, and aa zygotes sum to 1.)

(b) Assuming that the fitness of each genotype can be written as W_{AA}, W_{Aa}, and W_{aa}, derive the recursion equation for calculating $p(t+1)$.

3. (3 pts) The recursion derived in question 2 is a bit difficult to analyze. A simpler case of inbreeding is one in which only one of the two alleles is subject to inbreeding. Consider a diploid population with two alleles, A and a. The a allele is deleterious, meaning that it has a negative effect on fitness. This effect is stronger in homozygous aa individuals than in heterozygous Aa individuals, so the fitnesses can be arranged as $W_{\text{AA}} > W_{\text{Aa}} > W_{\text{aa}}$. Assume that all A gametes unite with a gamete chosen at random from the population, but an a gamete has a probability $1-f$ of uniting with a randomly chosen gamete, and a probability f of uniting with another a gamete. The recursion equation for this system is

$$p\,(t+1) = \frac{p^2\, W_{\text{AA}} + \left(1-\frac{f}{2}\right) p\,(1-p)\, W_{\text{Aa}}}{p^2\, W_{\text{AA}} + (2-f)\, p\,(1-p)\, W_{\text{Aa}} + (1-f)\,(1-p)^2\, W_{\text{aa}} + f(1-p)\, W_{\text{aa}}}$$

In the diploid natural selection model *without* inbreeding (studied in lecture), we found that when $W_{\text{AA}} > W_{\text{Aa}} > W_{\text{aa}}$, the equilibrium $\hat{p} = 0$ was unstable (the stability criterion was $W_{\text{Aa}} / W_{\text{aa}}$, which is greater than 1). For this system, what is the stability condition for the $\hat{p} = 0$ equilibrium? In terms of the genotype fitness, how large does the inbreeding coefficient f have to be for the $\hat{p} = 0$ equilibrium to be stable?

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