HOMEWORK # 10

(1) Here we will find the equilibria and determine their stability for two of the three predator-prey models derived in class. For both, $N(t)$ is the abundance of prey and $P(t)$ is the abundance of predators. The first model, the Lotka-Volterra predator prey model, assumes exponential growth of the prey population in the absence of predators, and a linear predator functional response:

$$\frac{dN(t)}{dt} = rN(t) - aN(t)P(t), \tag{1}$$
$$\frac{dP(t)}{dt} = baN(t)P(t) - mP(t). \tag{2}$$

The second model assumes logistic prey growth in the absence of predation, but still has a linear predator functional response:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) - aN(t)P(t), \tag{3}$$
$$\frac{dP(t)}{dt} = baN(t)P(t) - mP(t). \tag{4}$$

For each of these models, find *all* possible equilibria and evaluate the stability of each using the Jacobian matrix of partial derivatives. Keep in mind that stability requires that *all* eigenvalues of this matrix have negative real parts. If any eigenvalues have positive real part, the equilibrium is unstable. If the eigenvalues have 0 real part, the equilibrium is neutrally stable; that is, if you are perturbed off of the equilibrium, the variables will not return to the equilibrium but neither will they move away from it. Because these are two-dimensional systems, you might find it easier to compute the trace and determinant of the Jacobian: stability is guaranteed if the trace is negative and the determinant is positive. For each of the equilibria, state whether the equilibrium is stable or unstable. If stability depends on the parameter values, explain, in biological terms (based on the biological interpretation of each parameter), the conditions under which the equilibrium will be unstable versus stable.

(2) Consider the following simple disease model, where S is the fraction of the population that is susceptible to the disease, I is the fraction of the population that is infected, and R is the fraction of the

population that has recovered from the disease:

$$\frac{dS}{dt} = \mu - \beta SI - \mu S + \gamma R \tag{5}$$

$$\frac{dI}{dt} = \beta SI - \mu I - \rho I \tag{6}$$

$$\frac{dR}{dt} = \rho I - \gamma R - \mu R \tag{7}$$

Explain why this is a reasonable model for seasonal flu transmission, given the biological assumptions that have been made in deriving the model. Find the equilibria of this model. What condition must be met for the disease-free equilibrium to be unstable?