BIOM 300 – Mathematical Modeling in Biology

Course Instructor Clay Cressler
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Office Hours   Mondays 3:00-4:00pm in the Math Help Centre (Jeff 201)
         Fridays 3:00-4:00pm in the Math Help Centre (Jeff 201)
         Or by appointment
Course Text   *A Biologist's Guide to Mathematical Modeling in Ecology and*
         Evolution, by Sara Otto and Troy Day
Lecture logistics   Tuesday 9:30-10:30 in Jeffrey 128
         Thursday 8:30-9:30 in Jeffrey 126
         Friday 10:30-11:30 in Jeffrey 126
Final exam   To be announced

Learning Goals:
By the end of this class, students will be able to:
- describe the biological processes represented in a mathematical model and identify the key biological assumptions being made by the modeler;
- translate a biological description of a system into an appropriate mathematical model;
- analyze mathematical models both graphically (e.g., phase plane analysis) and numerically (e.g., simulation);
- calculate the stability of equilibria in both discrete and continuous time, for both single and multiple variable models.

Course Description:
Biological systems are incredibly complex, characterized by nonlinear interactions and many interacting units. Biologists want to understand these systems, from explaining species coexistence in the tropics, to predicting how a novel disease will spread, to managing the conservation of endangered species. However, the complexity of these systems can make it hard to achieve such understanding through experiment alone, as it can be hard to decide which biological processes to isolate experimentally.

For example, consider trying to predict the spread of an infectious disease. At the level of an individual host, the interaction between the immune system and the pathogen determines how many pathogens the person is carrying, how long they are transmitting disease, and whether their resistance to reinfection will be permanent (as with measles), temporary (as with flu), or non-existent (as with the common cold). All of these individual-level interactions will affect the processes of transmission and recovery. But there are other considerations as well. For example, the age of the sick person will affect her contact with others: babies and the elderly

don't get out much, compared to school-age or working-age people. But working-age people are much less likely to drool/sneeze/cough on a shared crayon box than your average kindergartener. Thus the age-structure (how many people are there of each age?) and contact structure (who comes in contact with who?) of the population may also affect transmission. Which of these interactions matters, in the sense that it has some detectable impact on disease spread? Answering that question experimentally would be difficult, to say the least. It might be more appropriate to say it would be impossible. This is where mathematical modeling can be a powerful tool for gaining biological insight.

Mathematical models formalize our assumptions about the processes driving biological systems as mathematical equations, allowing you to evaluate the dynamical consequences of different assumptions. This can provide the knowledge necessary to guide experiments, to shape policy, and to facilitate understanding. In this course, you will learn commonly used approaches for analyzing mathematical models in biology. Emphasis will be placed on the translation from biology to mathematical equations, and from mathematical analysis back to biological insight. As such, the mathematical techniques will be covered through their application to particular biological problems.

This course will draw on models of a range of biological systems, but with an emphasis on questions arising out of the consideration of ecological and evolutionary systems. Biological systems that will be studied in this course (we may not get to all of these topics!):
- single-species population growth
- species interactions (competition, predation)
- infectious disease spread
- natural selection in haploid and diploid species (e.g., population genetics)
- enzyme kinetics
- dynamics of populations structured by age, stage, space
- trait evolution (life history evolution, evolution of virulence)
- stochastic dynamics (birth-death, branching processes)

Mathematical topics that will be covered (again, we may not get to all of these!):
- linear and nonlinear single-variable differential and difference equations
- linear and nonlinear multivariate differential and difference equations
- linear stability analysis
- nondimensionalization
- sensitivity and elasticity analysis
- evolutionary invasion analysis

Assessment
Student grades will be based on a combination of **weekly homework** (30%), a **final exam** (30%), and a **group project** (40%). The final percentage mark will be converted to a letter grade according to official Queen's policy.

Weekly homework
A short (~3 question) homework set will be posted each Friday on the course website. Homework must be handed in, either in person or via email, by 12:00pm (noon) on the following Friday. There are 11 Fridays in the course, but only 9 of the homework assignments will count towards this mark. For this reason, *late homework will not be accepted.* Each homework assignment will be worth 10 points.

Final exam
There will be a **3-hr cumulative final exam** at some point during the exam period. Once I receive a date from the Exams Office, I will let you know. The final will cover all of the material from the course. No outside resources will be permitted for the final.

Group project
Mathematical modeling is a subject best learned by doing, like driving a car or cooking. **For this project, students will work in groups, choosing a biological question *related to host-parasite interactions* to investigate, formulating that question as a mathematical model, and then trying to answer that question using techniques learned during the course.** The restriction to host-parasite interactions is to facilitate topic selection within a broad field that leaves plenty of room to explore. However, this restriction is not absolute: groups of students wishing to work in a different area are welcome do so, but should contact me to help with topic choice. More information can be found on the Project page on Moodle or the course website**.** Briefly, the project is comprised of several components: a written project proposal, due to me on **3 October 2014** (10% of project mark); a rough draft of the written project report, due **7 November 2014** (20% of project grade); a peer evaluation form based on your review of another group's rough draft, due to me on **14 November 2014** (20% of project mark), and the final project report, due to me on **28 November 2014** (50% of project mark).

Policies:
Academic Integrity
The University has an official policy on academic integrity that will serve as the ultimate authority for arbitrating issues of academic integrity. The main points that are relevant for this course are as follows: on homework and the project, any and all resources are fair game but they *must be acknowledged*. I expect that every assignment will be turned in with an acknowledgements section explicitly referencing those resources. Even a statement as simple as, *"Jane Smith and I had a conversation about this homework after class"* would be sufficient, if that was genuinely the only resource that was used. No outside resources will be allowed for the final exam.

Plagiarism
Plagiarism, in this course, will be defined as the improper use of another person's words or ideas without proper citation. Of course, reading others' work as you do

the research for your project will help inform your understanding. This is how science is done! However, I *strongly discourage* the use of direct quotation, as this is essentially never done in scientific writing. Instead, *synthesize what you have learned from your research and state it in your own words.* Note that you do not need to cite every statement! Any statement, whether biological or mathematical, that you could reasonably expect someone in the class to know does not need to be cited. When in doubt, err on the side of over-referencing versus under-referencing, and I am happy to provide further clarification when necessary.

Citation Style
Citations should be in the AMS style.

Students with Disabilities
Any student with disabilities should contact Disability Services. I am happy to help facilitate that interaction if necessary. That office has trained clinicians who will diagnose and document the disability, and can decide what accommodations are best for each individual. I will follow whatever course they recommend.

Email Policy
Email can be a pernicious timesink. Because of that, I check email at designated times throughout the day, and may not reply to your email immediately. During the week, I promise to reply to any email within 24 hours. Any email sent after 6:00pm on Friday, however, may not receive a reply until Monday morning, so plan accordingly!

Course Schedule
There is no fixed calendar for this course. I will spend as much time on each topic as is necessary, and we will go as far as we can through the material presented in Otto and Day. Though it may not look it, the list of topics below is pretty ambitious. I give the order topics will be covered in, with corresponding readings in Otto and Day for further reading/reference.

Topic	Textbook readings
Introduction to mathematical biology	Ch. 1, Epilogue
How to construct a model	Ch. 2, 3
Single-species population models in discrete and continuous time	Chs. 3.2, 3.3, 4.2, 4.3, 5, 6
Linear multivariable models in discrete and continuous time	Chs. 4.4, 7, 9.2, 10
Nonlinear multivariable models in discrete and continuous time	Ch. 3.4, 3.5, 4.4, 8, 9.3, 11.4

Evolutionary invasion analysis   Ch. 12

Stochastic models   13.3-13.5, 14

Supplementary Resources
Some books that you might find helpful at various points throughout the course are

Caswell, H. 2001. *Matrix Population Models.* Sinauer Associates, Sunderland, MA.

Edelstein-Keshet, L. 2005. *Mathematical Models in Biology*. Society for Industrial and Applied Mathematics, Philadelphia, PA.

Keeling, M. J. and P. Rohani. 2008. *Modelling Infectious Disease.* Princeton University Press, Princeton, NJ.

Kot, M. 2001. *Elements of Mathematical Ecology*. Cambridge University Press, Cambridge, UK.

Murray, J. D. 2001. *Mathematical Biology, Part I: An Introduction*. Springer, New York, NY.

At various points in the course I may also provide readings from the primary mathematical biology literature. Papers will be posted to on the course website at that time.